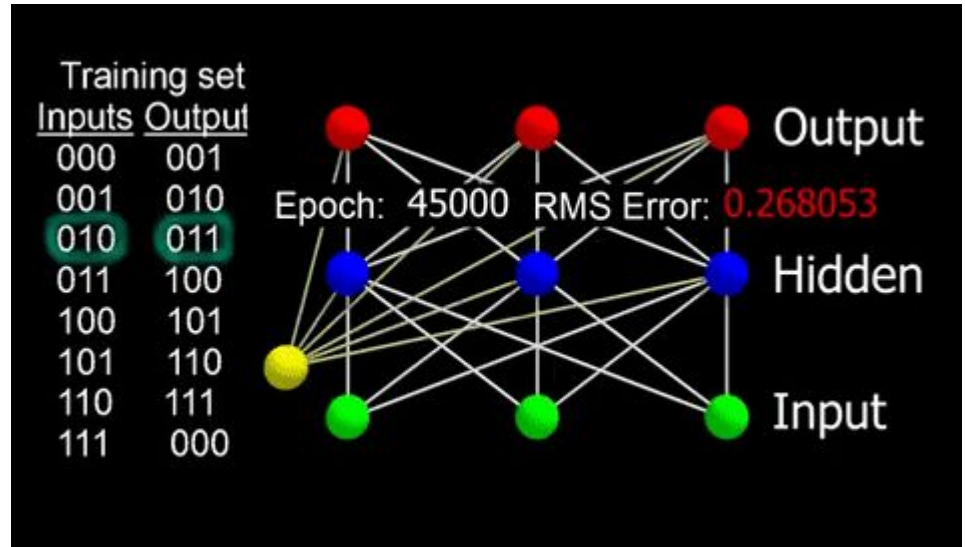


Learning representations by back-propagating errors

Group 05

Madhushani H. K.	E/15/209
Maliththa K.H.H.	E/15/220
Muthucumarana P.N.N.	E/15/233
Sankalpana W.A.P.C.	E/15/325



The Authors

- David E. Rumelhart
- *Stanford University*
- Geoffrey E. Hinton
- *University of Toronto*
- Ronald J. Williams
- *Northeastern University*



in

- 1986

What does mean by 'Backpropagation' ?

- The term *backpropagation* strictly refers only to the algorithm for computing the gradient.
- In machine learning, **backpropagation** is a widely used algorithm for training feedforward neural networks.
- Generalizations of backpropagation exists for other Artificial Neural Networks(ANNs)
- These classes of algorithms are all referred to generically as "**backpropagation**".

- The practice of fine-tuning the weights of a neural net
 - based on the error rate obtained in the previous epoch
- Activations of the input units are propagated forward to output layer through the connecting weights.

Why 'Backpropagation' ?

- Computes the gradient of the loss function with respect to the weights of the network for a single input–output example
- Does so efficiently, unlike a naive direct computation of the gradient
- Proper tuning of the weights ensures lower error rates
- This efficiency makes it feasible to use gradient methods
- Avoid redundant calculations of intermediate terms in the chain rule
- Image recognition and speech recognition

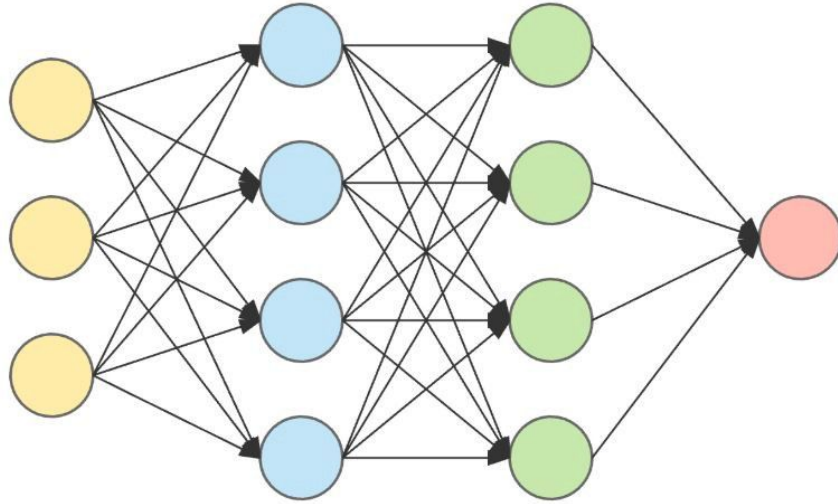
Back-propagating error

- Backpropagation, short for "backward propagation of errors," is an algorithm for supervised learning of ANNs using gradient descent.
- Backpropagation is analogous to calculating the delta rule for a multilayer feedforward network.

backpropagation requires three things:

- Dataset
- A feedforward neural network,
- An error function

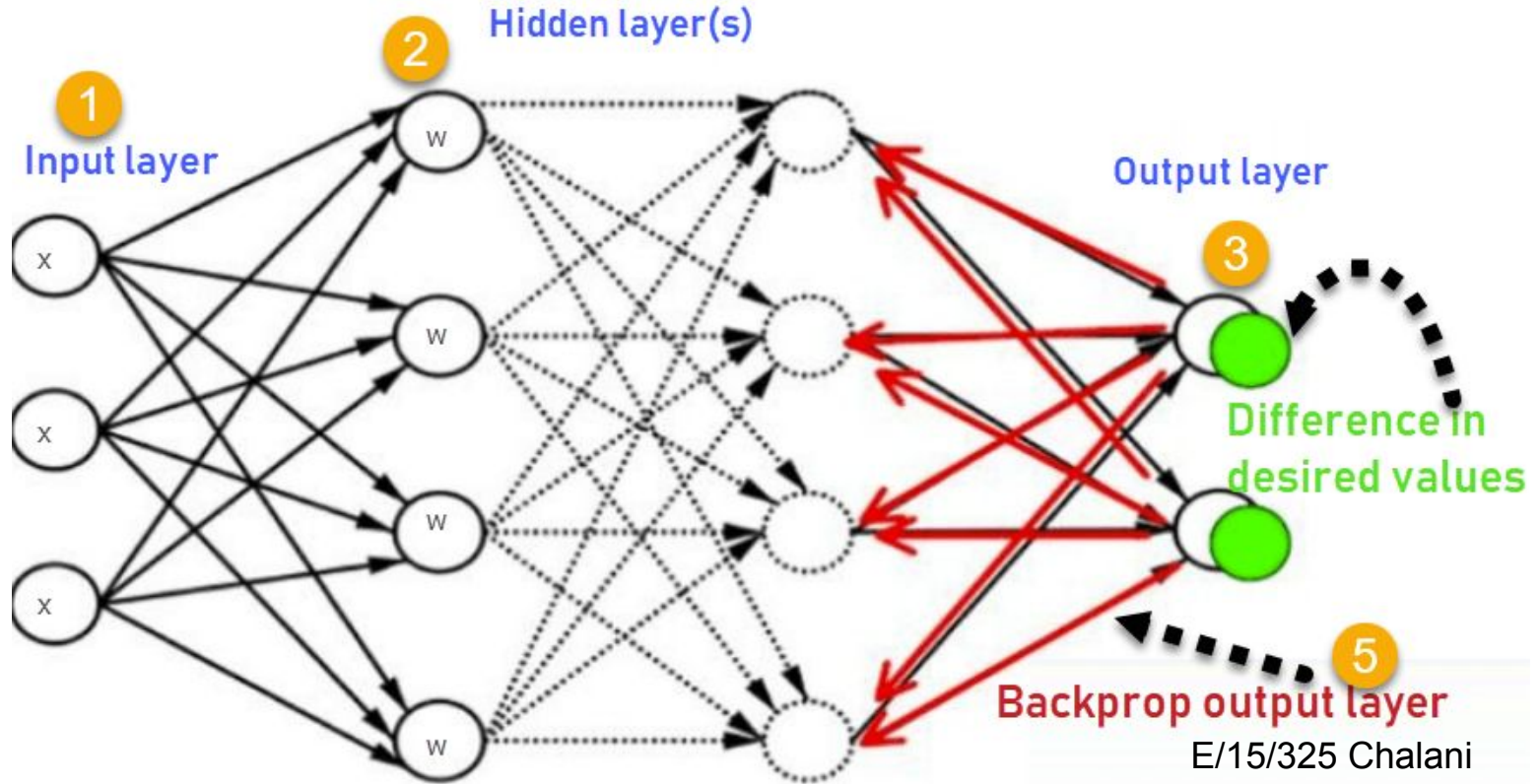
$$\underline{x} = \begin{pmatrix} x_0 \\ \dots \\ x_j \\ \dots \end{pmatrix}$$



$$\underline{y} = \begin{pmatrix} y_0 \\ \dots \\ y_j \\ \dots \end{pmatrix}$$

How Backpropagation Works

<https://www.guru99.com/backpropagation-neural-network.html>



Back-Propagation Learning Procedure

- The total input x_j , to unit j

y_i - output of unit i connected to unit

w_{ji} - weight connecting unit i to unit j

$$x_j = \sum_i y_i w_{ji}$$

- Resulting value is passed through a sigmoid function

$$y_j = \frac{1}{1 + e^{-x_j}}$$

- Aim is to find a set of weights that gives the output vector produced by the network,
- same as the desired output vector.
- The total error comparing the actual and desired output vector:

$$E = \frac{1}{2} \sum_c \sum_j (y_{j,c} - d_{j,c})^2$$

c - index over cases (input-output pairs),

j - index over output units

y - actual state of an output unit

d - desired state.

- To minimize E by gradient descent → partial derivative of E

$$E = \frac{1}{2} \sum_c \sum_j (y_{j,c} - d_{j,c})^2$$

Let's Differentiate the above equation,

$$\frac{\partial E}{\partial y_j} = y_j - d_j$$

Let's apply the chain rule to compute $\frac{\partial E}{\partial x_j}$

$$\frac{\partial E}{\partial x_j} = \frac{\partial E}{\partial y_j} \frac{dy_j}{dx_j}$$

$$y_j = \frac{1}{1 + e^{-x_j}}$$

Let's differentiate the above equation to get the value of dy_j/dx_j , and substituting gives

$$\frac{\partial E}{\partial x_j} = \frac{\partial E}{\partial y_j} y_j (1 - y_j)$$

Let's compute how the error by changing these states and weights. For a weight w_{ji} , from i to j the derivative is

$$\begin{aligned} \frac{\partial E}{\partial w_{ji}} &= \frac{\partial E}{\partial x_j} \frac{\partial x_j}{\partial w_{ji}} \\ &= \frac{\partial E}{\partial x_j} y_i \end{aligned}$$

For the output of the i th unit the contribution to $\partial E/\partial y_i$;

$$\frac{\partial E}{\partial x_j} \frac{\partial x_j}{\partial y_i} = \frac{\partial E}{\partial x_j} w_{ji}$$
$$\frac{\partial E}{\partial y_i} = \sum_j \frac{\partial E}{\partial x_j} w_{ji}$$

Ways of using $\frac{\partial E}{\partial w}$

1. Change the weights after every input-output case.
2. Accumulate $\frac{\partial E}{\partial w}$ over all the input-output cases before changing the weights.
 - An alternative scheme
3. Change each weight by an amount proportional to the accumulated $\frac{\partial E}{\partial w}$
 - The simplest version of gradient descent

$$\Delta w = -\varepsilon \frac{\partial E}{\partial w}$$

An improved version

$$\Delta w(t) = -\varepsilon \frac{\partial E}{\partial w(t)} + \alpha \Delta w(t - 1)$$

- t - incremented by 1 for each sweep through the whole set of input-output cases
- α - is an exponential decay factor between 0 and 1

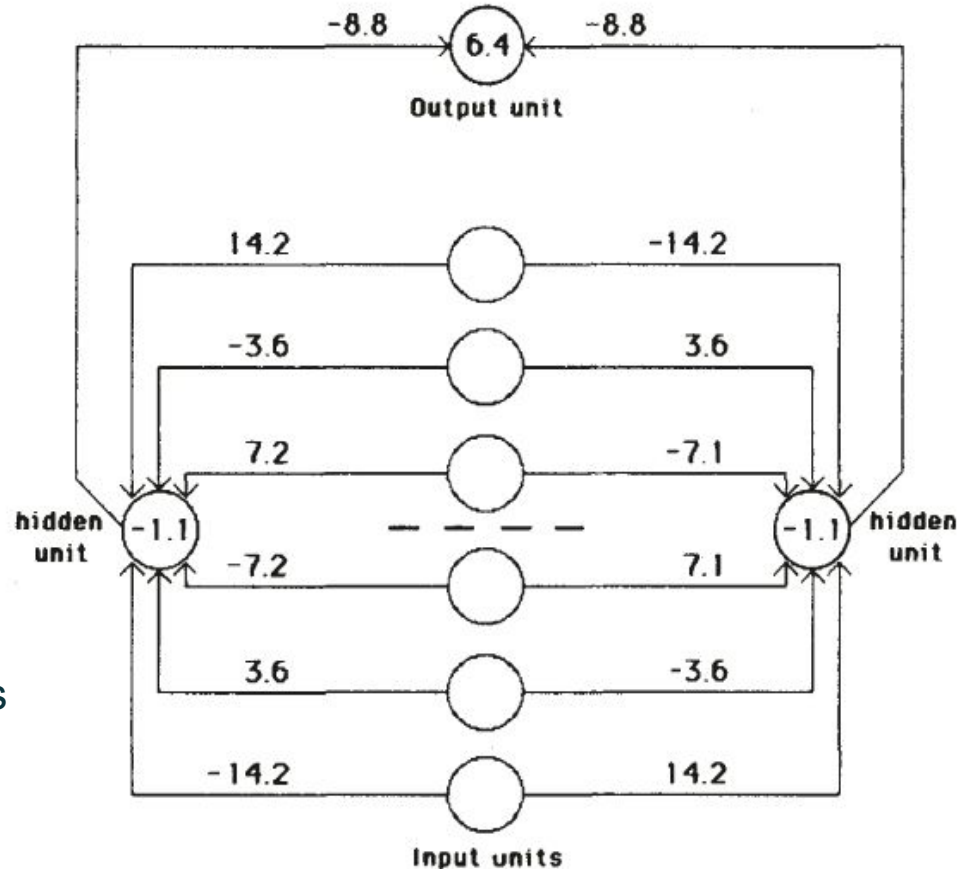
Detection of symmetry

use an intermediate layer

- weights that are symmetric about the middle of the input vector are equal in magnitude and opposite in sign.

Advantage:

- Both hidden units will receive a net input of 0 from the input units
- Weights on each side of the midpoint are in the ratio 1 : 2 : 4.

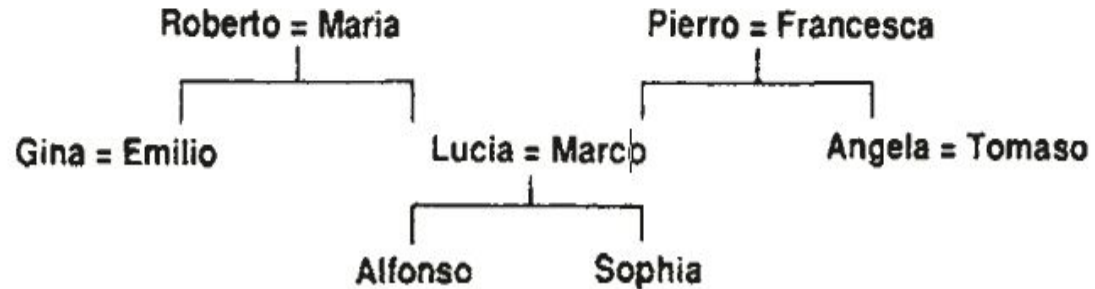
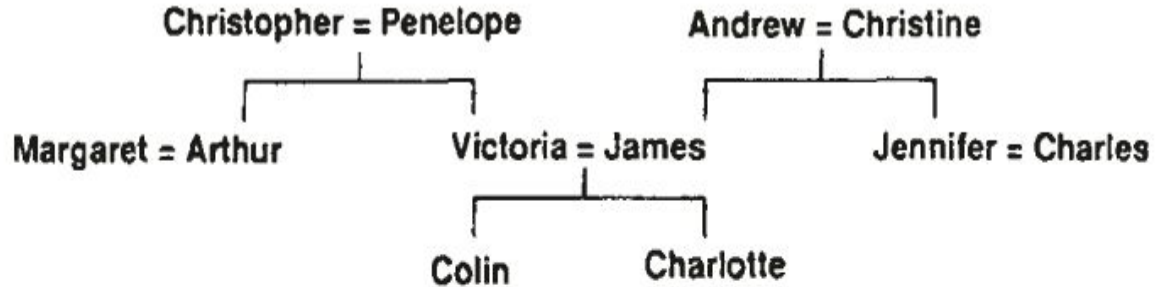


Storing the information in family trees

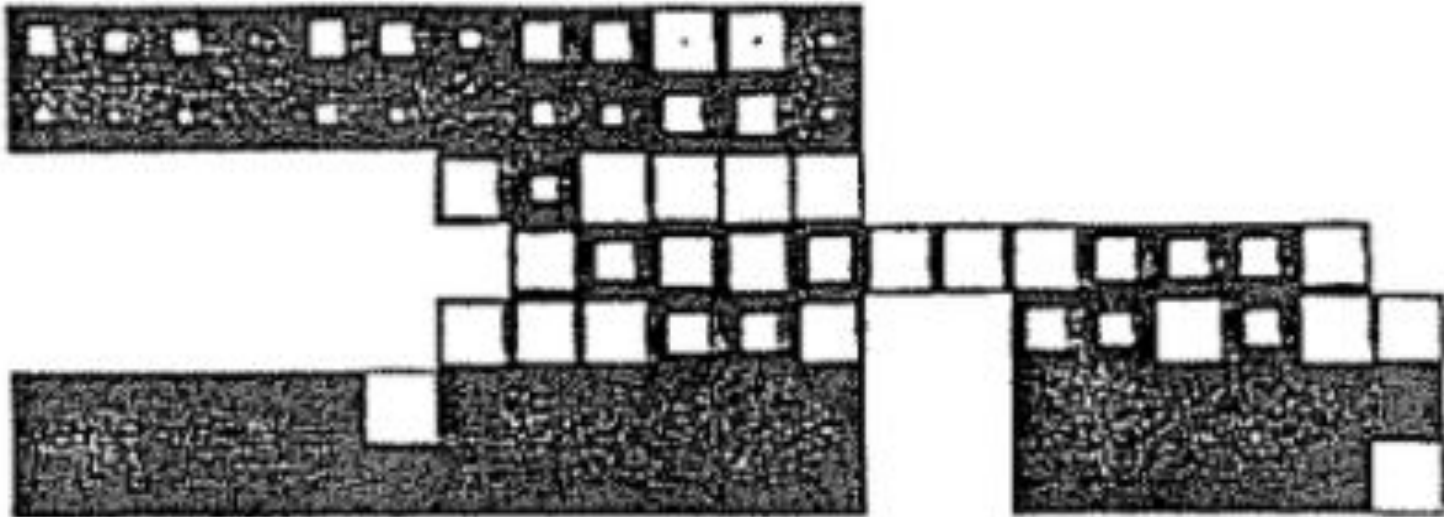
Set of propositions using the 12 relationships:

son, daughter, nephew, niece,
father, mother, uncle, aunt,
brother, sister, husband, wife

- (colin has-father james)
- (colin has-mother victoria)
- (james has-wife victoria)
- (charlotte has-brother colin)
- (victoria has-brother arthur)
- (charlotte has-uncle arthur)

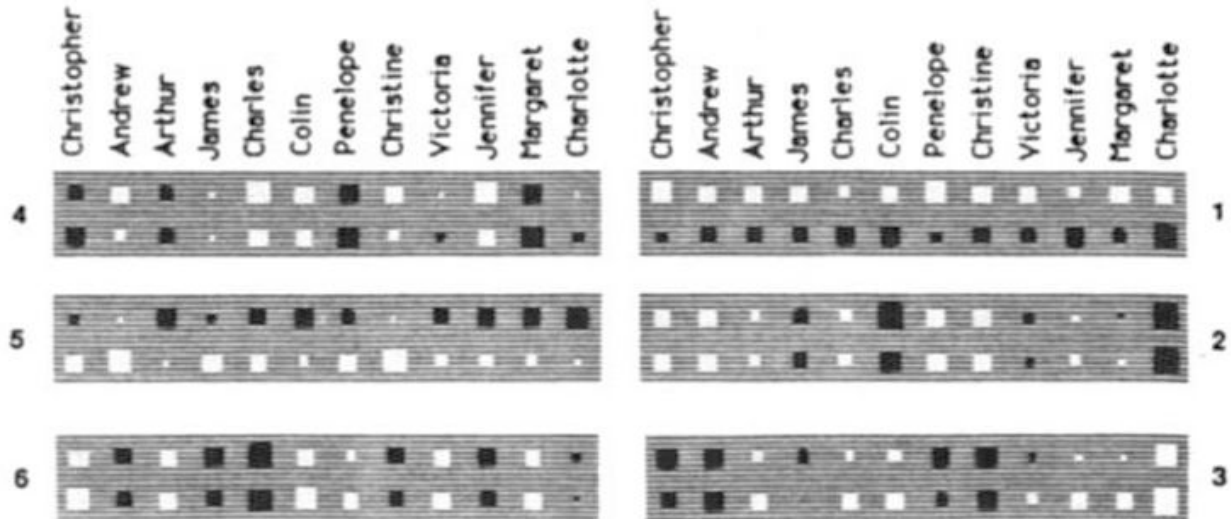


Network



Receptive fields

- Trained on 100 of the 104 possible triples
 - White rectangles - excitatory weights
 - black rectangles - inhibitory weights
 - area of the rectangle - encodes the magnitude of the weight.

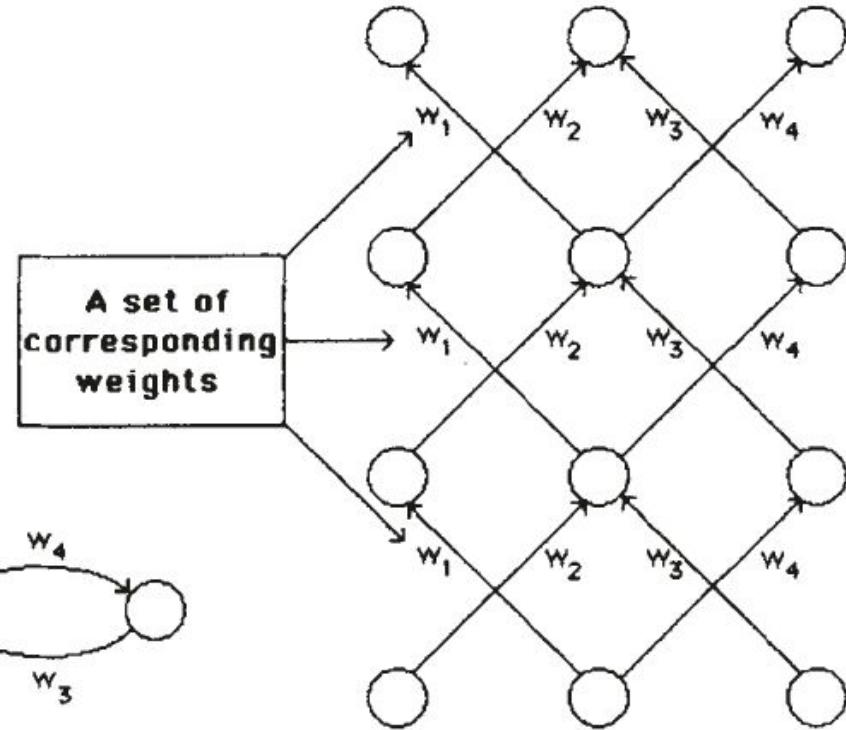
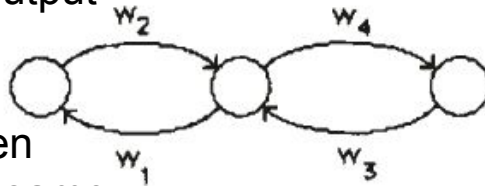


The layered network

Layered nets \rightarrow Iterative nets.

Two complications arise in performing the mapping:

- Needs to store the history of output states of each unit
- corresponding weights between different layers must have the same value



Thank You!

Q & A